A Survey of algebras based on graphs

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Varieties generated by algebras resulting from (both directed and undirected) graphs in various ways were studied by several groups of mathematicians. Our group, including R. Freese, P. Jipsen, M. Maróti, P. Marković and R. McKenzie, investigated the groupoids resulting from directed graphs with loops (i.e., reflexive binary relations) by setting $xy = x$ whenever $x \to y$, and $xy = y$ otherwise. These are precisely the conservative groupoids (such that every nonempty subset is a subgroupoid). Most attention was paid to the varieties generated by algebras resulting from tournaments, ordered sets and equivalence relations. In each case we were trying to decide if the variety is finitely based, what are its subdirectly irreducible algebras, and what can be said about the lattice of subvarieties. The last of these three varieties is quite small, while for the first two varieties many questions remain unsolved.

Also, we do not know much about the variety generated by all conservative groupoids. (We know that the variety is not finitely based.)